1 a $f(x) = \sin 2x, \ x \in [0, 2\pi]$
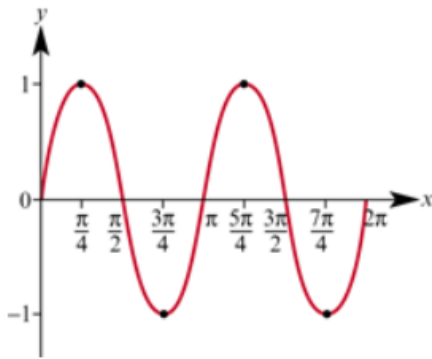
The transformation from the graph of $g(x) = \sin x$ is a dilation from the y axis of factor $\dfrac{1}{2}$.
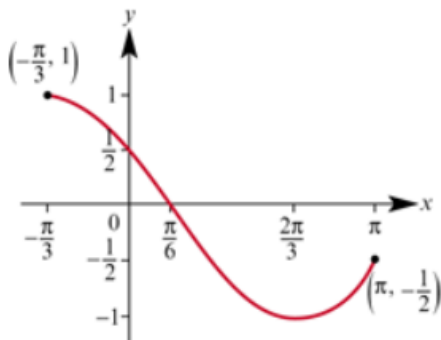

b

$$f(x) = \cos\left(x + \frac{\pi}{3}\right), \ x \in \left[-\frac{\pi}{3}, \ \pi\right]$$

The transformation from the graph of $g(x) = \cos x$ is a translation of $\dfrac{\pi}{3}$ to the left.

$$f\left(\frac{-\pi}{3}\right) = \cos 0 = 1$$

$$f(0) = \cos\frac{\pi}{3} = \frac{1}{2}$$

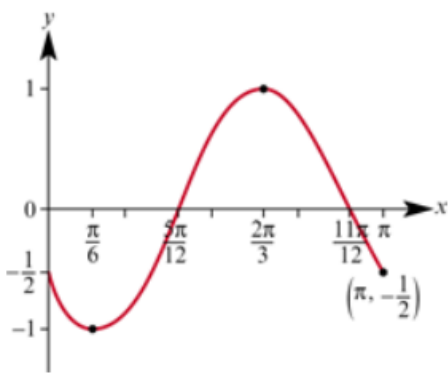$$f(\pi) = \cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

c

$$f(x) = \cos\left(2\left(x + \frac{\pi}{3}\right)\right), \ x \in [0, \ \pi]$$

The transformations from the graph of $g(x) = \cos x$ are a dilation from the y axis of factor $\dfrac{1}{2}$ and a translation of $\dfrac{\pi}{3}$ to the left.

$$f(0) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$f(\pi) = \cos\left(\frac{8\pi}{3}\right) = -\frac{1}{2}$$

d

$f(x) = 2\sin(3x) + 1,\ x \in [0,\ \pi]$

The transformations from the graph of $g(x) = \sin x$ are a dilation from the y axis of factor $\dfrac{1}{3}$, a dilation from the x axis of factor 2 and a translation of 1 in the positive direction of the y axis.
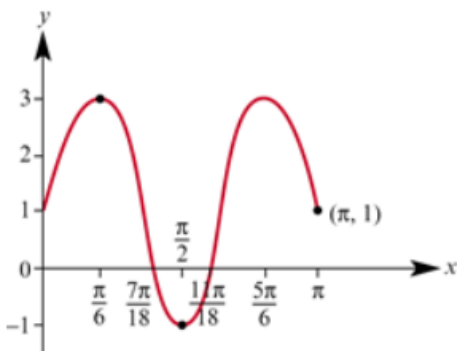
To find x axis intercepts for $f(x)$, solve $f(x) = 0$

i.e. $2\sin(3x) + 1 = 0,\ x \in [0,\ \pi]$

$\therefore \sin(3x) = -\dfrac{1}{2},\ 3x \in [0,\ 3\pi]$

$\therefore 3x = \dfrac{7\pi}{6},\ \dfrac{11\pi}{6}$

$\therefore x = \dfrac{7\pi}{18},\ \dfrac{11\pi}{18}$

$f(0) = 1,\ f(\pi) = 2\sin(3\pi) + 1$
$$= 1$$

e

$f(x) = 2\sin\left(x - \dfrac{\pi}{4}\right) + \sqrt{3},\ x \in [0,\ 2\pi]$

The transformations from the graph of $g(x) = \sin x$ are a dilation from the x axis of factor 2, a translation of $\dfrac{\pi}{4}$ to the right and a translation of $\sqrt{3}$ in the positive direction of the y axis.

$f(0) = 2\sin\left(-\dfrac{\pi}{4}\right) + \sqrt{3}$

$ = -2\sin\left(\dfrac{\pi}{4}\right) + \sqrt{3}$

$ = \sqrt{3} - \sqrt{2}$

$f(2\pi) = 2\sin\left(\dfrac{7\pi}{4}\right) + \sqrt{3}$

$ = \sqrt{3} - \sqrt{2}$

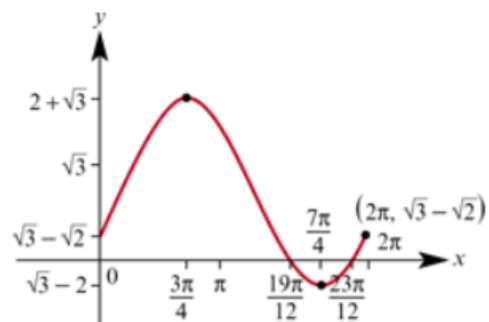To find x axis intercepts for $f(x)$, solve $f(x) = 0$

i.e. $2\sin\left(x - \dfrac{\pi}{4}\right) + \sqrt{3} = 0$, $x \in [0, 2\pi]$

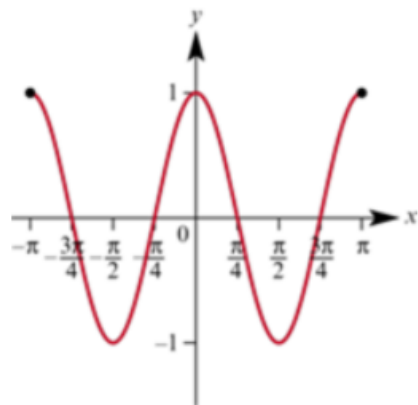$\therefore \sin\left(x - \dfrac{\pi}{4}\right) = \dfrac{-\sqrt{3}}{2}$,

$x - \dfrac{\pi}{4} \in \left[-\dfrac{\pi}{4}, \dfrac{7\pi}{4}\right]$

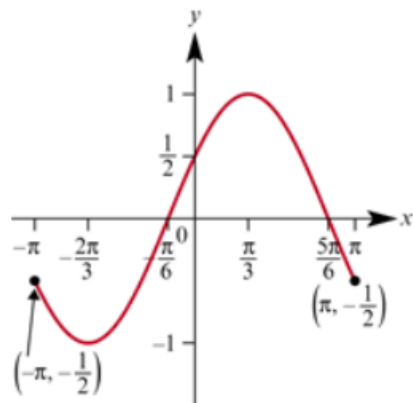$\therefore x - \dfrac{\pi}{4} = \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$
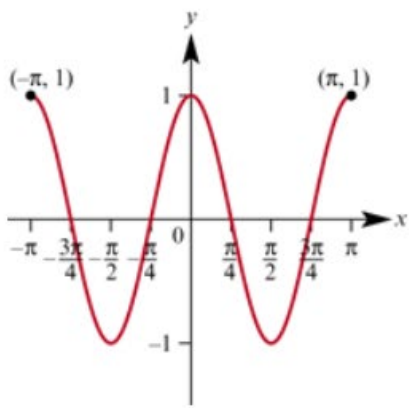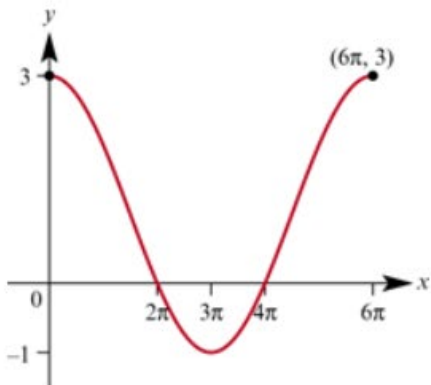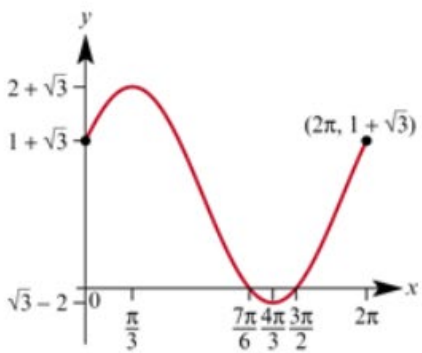
$\therefore x = \dfrac{19\pi}{12}, \dfrac{23\pi}{12}$
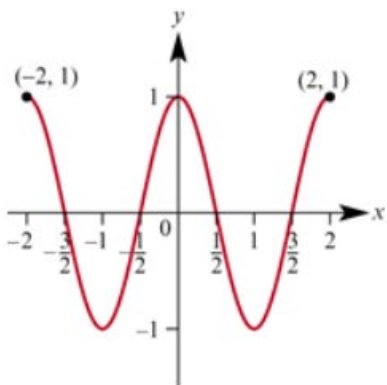

f

2 a

b

$(-\pi, 1)$ $(\pi, 1)$

y

1

x

$-\pi$ $-\dfrac{3\pi}{4}$ $-\dfrac{\pi}{2}$ $-\dfrac{\pi}{4}$ 0 $\dfrac{\pi}{4}$ $\dfrac{\pi}{2}$ $\dfrac{3\pi}{4}$ π

-1

c

y

$(6\pi, 3)$

3

0 2π 3π 4π 6π x

-1

d

y

$2 + \sqrt{3}$

$1 + \sqrt{3}$ $(2\pi, 1 + \sqrt{3})$

$\sqrt{3} - 2$ 0 $\dfrac{\pi}{3}$ $\dfrac{7\pi}{6}$ $\dfrac{4\pi}{3}$ $\dfrac{3\pi}{2}$ 2π x

e

$(-2, 1)$ $(2, 1)$

y

1

-2 $-\dfrac{3}{2}$ -1 $-\dfrac{1}{2}$ 0 $\dfrac{1}{2}$ 1 $\dfrac{3}{2}$ 2 x

-1

f

$(-12, 1)$ $(12, 1)$

y

1

-12 -9 -6 -3 0 3 6 9 12 x

-1